MTL 106 (Introduction to Probability and Stochastic Processes)

II Semester 2016-17

Tutorial Sheet 2

Random Variable

1. Consider a probability space (Ω, F, P) with Ω = {0, 1, 2}, F = {Φ, {0}, {1, 2}, Ω}, P({0}) = 0.5 = P({1,2}). Give an example of a real valued function on Ω that is not a random variable. Justify your answer.
2. Do the following functions define distribution functions.
   1. F(x) = ()tan-1x,
3. Consider the random variable X that represents the number of people who are hospitalized or die in a single head-on collision on the road in front of a particular spot in a year. Cumulative distribution function of X is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | F (x) | 0.250 | 0.546 | 0.898 | 0.932 | 0.955 | 0.972 | 0.981 | 0.989 | 0.995 | 0.998 | 1.000 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

Find P(X = 10) and P(X ≤ 5 / X > 2)

1. For what values of α, p does the following function represent a probability mass function pX(x) = αpx, x = 0. 1, 2, …Prove that the random variable having such a probability mass function satisfying the following memoryless property P(X > a + s / X > a) = P(X ≥ s)
2. Let X be a random variable such that P(X = 2) = and its distribution function is given by
3. Find α, 𝞫 if 2 is the only jump discontinuity of F.
4. Compute P(X < 3 / X ≥ 2)
5. A student arrives to the bus stop at 6:00 AM sharp, knowing that the bus will arrive in any moment, uniformly distributed between 6:00 AM and 6:20 AM.
   1. What is the probability that the student must wait more than five minutes?
   2. If at 6:10 AM the bus has not arrived yet, what is the probability that the student has to wait at least five more minutes?
6. Accidents in Delhi roads involving Blueline buses obey Poisson process with 9 per month of 30 days. In a randomly chosen month of 30 days.
   1. What is the probability that there are exactly 4 accidents in the first 15 days?
   2. Given that exactly 4 accidents occurred in the first 15 days, what is the probability that all the four occurred in the last 7 days out of these 15 days?
7. The time to failure of certain units is exponentially distributed with parameter λ. At time t = 0, n identical units are put in operation. The units operate, so that failure of any unit is not affected by the behavior of the other units. For any t > 0, let Nt be the random variable whose value is the number of units still in operation time t. Find the distribution of the random variable Nt
8. Suppose that f and g are density function and that 0 < λ < 1 is a constant. (a) Is λ f + (1 - λ)g a probability density function? (b) Is fg a probability density function? Explain.
9. Let X be a random variable with cumulative distribution function given by:

Determine the cumulative discrete distribution functions Fd and one continuous Fc such that: FX(x) = αFd(x) + βFc(x)

1. Suppose that duration (measured in minutes) of a telephone conversation between two persons is a random variable X with cumulative distribution function

Given that conversation has been going on for 20 minutes, compute the probability that it continues for at least another 10 minutes

1. Suppose that the life length of two electronic devices say D***1*** and D***2*** have normal distributions N(40, 36) and N(45, 9) respectively. (a) If a device is to be used for 45 hours, which device would be preferred? (b) If it is to be used for 42 hours which one should be preferred.